M.C. Escher: Artist and Mathematician

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M.C. Escher is an artist who incorporates mathematical themes into his prints, and by examining his prints, one can gain a new appreciation for the mathematics behind everyday things as well as the math behind more fantastical images.

To give context to Escher’s work, one should know a bit about the artist himself. Escher is a Dutch artist who lived between the years 1898-1972 and is famous primarily for his graphic prints (Bool, 6). Escher grew up in Arnhem, which is a city in the Netherlands, and attended high school there. He then went to the School for Architecture and Decorative Arts in Haarlem for college, where he had S. Jessurun de Mesquita as a graphic arts teacher, who helped Escher decide to become a graphic artist (Bool, 6). Escher would go on to use a variety of graphic printing methods in his work, such as lithography, woodcuts, mezzotints to create these graphic prints. Interestingly, Escher actually did very poorly in both high school and college, and it was said about Escher, in reference to his school years, “Later, when his prints were particularly admired by mathematicians, he would exaggerate by claiming that he never got the hang of it” (Bool, 15).

After Escher graduated college, he traveled around Spain and Italy for about two years, where the surrounding landscape had a big influence on the landscapes of Escher’s work (Bool, 21-30). Many of these landscapes are brilliantly constructed cityscapes and mountain sides that look more precise than the places they were based on. During this time and throughout the rest of his life, Escher worked on commissioned pieces as well as his personal work. One of Escher’s more famous commissions is the Metamorphosis mural, which on display in a post office (Bool, 134). When looking at the body of Escher’s more personal work, Escher explores different mathematical through his work throughout his life. Some of these themes are geometric forms,
tessellations/metamorphoses, pictorial representations of infinity, Mobius strips, and impossible objects (Ernst, 21 and Bool, 135).

In Escher’s work, he often makes use of perspective in order to give the objects in his pictures a sense of realism, or like it is an object or scene that can exist in real life. In three-dimensional perspective drawings, an object can be drawn as taking place on a grid, with objects of the same size shrinking in size but maintaining the same proportions as they approach the vanishing point in order to give the appearance of an object being the same size but receding into the distance (Powell, 8). The vanishing point in a drawing is the point where an object would recede so far into the distance as to be become invisible, which is why it’s called the vanishing point. One of Escher’s illustration that works as a good example of the geometry in perspective is the illustration called *Concentric Rinds (Concentric Space Filling)* (Thé, figure 292), which is a fantastic print that shows circles composed of connected rings inside other circles composed of rings. The entire illustration must have taken ages to execute correctly to give the appearance of an actual object in space while maintaining that level of complexity.

In Escher’s studies for the illustration *Rippled Surface* (Thé, figure 176-178), one can gain an appreciation of the mathematics involved in even something so simple as a ripple on the surface of water. In the illustration studies, you can see the ripples being mapped out as a disk being tilted at an angle, so if the disk was real it could be viewed as a perfect circle from a top down view. In the final image, which is depicted as a rough draft on the same page as the ripple studies, you can see the precise angle of distortion of the reflected image.

Related to these drawing using traditional perspective, Escher also bends the rules of perspective in order to get impossible objects, like the Penrose triangle. An impossible object is an object that can be represented on a two-dimensional plane but not on a three-dimensional
plane. I have drawn a Penrose triangle and an impossible cube to give as examples of impossible shapes to avoid confusion by what was meant. One print of Escher’s that demonstrates the way he uses this idea is called *Waterfall* (Ernst, figure 200) which shows two interconnected impossible triangles by showing a waterwheel making water flow uphill through a series of canals whose architecture cannot exist because it breaks the illusion of receding space. Escher also used an impossible cube as a basis for the architecture in the print *Belvedere* (Ernst, figure 186), in which a straight background line will cross over a line in the foreground then reconnect with a point in the background, breaking the illusion of space and making the form impossible to exist in real life.

Another mathematical theme that runs through Escher’s work are tessellations. Tessellations are patterns that leaves no gaps between the shapes, with a good example being a checker board or tile floor, because they are formed by perfectly fitting interlocking squares. While Escher has done many things with traditional tessellations, such as *Regular Division of the Plane Drawing #20* (Thé, figure 149). There is actually a theorem based on Escher’s work involving tessellations with hexagons, and can be proved using tessellations, called the “Escher Theorem” that goes as follows:

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i. Let $A'B'C'$ be an equilateral triangle and $B$ any point. Let $C$ be the point such that $A'B = A'C$ and $\angle CA'B = 120^\circ$. Let $A$ be such that $B'A = B'C$ and $\angle CB'A = 120^\circ$. Then $C'A = C'B$ and $\angle AC'B = 120^\circ$.

ii. Congruent copies of hexagon $AC'BA'CB'$ can be used to tessellate the plane.
iii. *The line AA', BB', CC' are concurrent.* 

(Bogomolny, N.pg.).

Interestingly, while Escher frequently used the theorem in his work and in his notes, he was not the person who proved it, but he did arrive at his conclusions independently (Rigby, 242-243).

In Escher’s late life he explored the themes of infinity in his drawings. One good example of this is shown in his print *Circle Limit III* (Ernst, figure 244). In the print, Escher manages to circumscribe an infinite number of tessellated fish into a circle by having the fish get infinitely smaller as they approach the edges, but never surpassing the shape. M.C. Escher based his print off an illustration found in a book by H.S.M Coxeter that shows an infinite plane contained within a circle (Ernst, 108). This way of representing infinity reminds me of a hypothetical situation in which a person, whose start point is always point A, can only move half way from point A to point B each time they move, they can take an infinite number of steps and still never reach point B. The reason the print reminds me of that situation is because by using the same logic, if a person were to draw an object that filled half the space between point A and point B every time they had drawn an object then they would have drawn an infinite number of objects without filling the space.

In conclusion, M.C. Escher is an artist whose work’s mathematical themes brings to the forefront the beauty of math and reminds us of its place in the world around us as well as in the worlds of our imagination.


